MAGNETOAERODYNAMICS IN JAPAN

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Summary—Recent theoretical researches in magnetoaerodynamics carried out in Japan are reported. They are mostly concerned with flows of electrically conducting fluids past two- and three-dimensional bodies, such as circular cylinders and spheres, in the presence of a uniform magnetic field. Small-disturbance theories, including the Stokes and Oseen type approximations for viscous flows and slender-body theory for inviscid flows, are developed. Peculiar behaviour of magnetohydrodynamic wakes accompanying bodies placed in uniform magnetic and flow fields is investigated. Flows of conducting fluids past magnetized bodies are considered. Jets and boundary layers are also studied.

1. INTRODUCTION

This paper reports on the results of theoretical research in the field of magnetoaerodynamics recently performed in Japan. Emphasis will be placed on the study of magnetohydrodynamic flows of electrically conducting fluid past bodies, with special reference to the forces acting on them. Thus, the works mainly in the field of astrophysics or thermonuclear research are not mentioned, while the studies concerning boundary layers and jets are only briefly touched upon.

2. THE STOKES TYPE APPROXIMATION FOR MAGNETOHYDRODYNAMICS

Chester[1] was the first to consider the magnetohydrodynamic flow of an electrically conducting fluid past a body in the presence of magnetic field. He considered the slow motion of a sphere in the direction of a uniform magnetic field. He obtained the formula for the drag D in the form:

$$D = D_S \left\{1+\frac{3}{8}H+\frac{7}{960}H^2-\frac{43}{7680}H^3+(H^4)\right\} \tag{2.1}$$

where D_S is the classical Stokes drag, and H is the Hartmann number. Thus

$$D_S = 6\pi\varrho\nu\, aU, \tag{2.2}$$

$$H = (\sigma/\varrho\nu)^{1/2}B_\infty a \tag{2.3}$$

where U is the velocity of the sphere, a its radius, ϱ, ν and σ are respectively the density, the kinematic viscosity and the electrical conductivity of the fluid, and B_∞ is the magnetic induction of the uniform magnetic field.

In the rationalized MKS system of units, the basic equations of motion for an incompressible, viscous and electrically conducting fluid in the presence of magnetic field can be written in the forms:

$$\varrho(\boldsymbol{V}\cdot\text{grad})\boldsymbol{V} = -\text{grad}\, p+\varrho\nu\Delta\boldsymbol{V}+\boldsymbol{j}\times\boldsymbol{B} \tag{2.4}$$

$$\text{div}\ \boldsymbol{V} = 0 \tag{2.5}$$

$$\text{curl}\,\boldsymbol{E} = 0, \quad \text{curl}\,\boldsymbol{H} = \boldsymbol{j}, \quad \text{div}\,\boldsymbol{B} = 0 \tag{2.6}$$

$$\boldsymbol{B} = \mu\boldsymbol{H} \tag{2.7}$$

$$\boldsymbol{j} = \sigma(\boldsymbol{E}+\boldsymbol{V}\times\boldsymbol{B}) \tag{2.8}$$

where $\boldsymbol{V}$ is the velocity, p the pressure, $\boldsymbol{E}$ the electric field, $\boldsymbol{H}$ the magnetic field, $\boldsymbol{B}$ the magnetic induction, $\boldsymbol{j}$ the electric current density, and μ the permeability. Eqs. (2.4), (2.5), (2.6) and (2.8) are the equation of motion, the equation of continuity, Maxwell's equations and Ohm's law respectively. For magnetohydrodynamic approximation, the equations div $\boldsymbol{D} = \varrho e$ and $\boldsymbol{D} = \varepsilon\boldsymbol{E}$ are not needed.

Chester made use of the fact that the magnetic field is not disturbed by the motion of the sphere, for small magnetic Reynolds number. Thus, the effect of the magnetic field appears only as the Lorentz force $\boldsymbol{j}\times\boldsymbol{B}$ in the equation of motion (2.4), such that

$$\boldsymbol{j}\times\boldsymbol{B} = \sigma(\boldsymbol{V}\times\boldsymbol{B}_\infty)\times\boldsymbol{B}_\infty$$

$\boldsymbol{B}_\infty$ being the undisturbed magnetic field. Then the velocity $\boldsymbol{V}$ and pressure p can be expressed in forms quite similar to the Oseen approximation for the ordinary non-conducting fluid, in spite of the fact that the Stokes type approximation is employed for the equation of motion (2.4). The drag D is greater than D_s. This may be naturally expected in view of the fact that the magnetic lines of force would behave just like stretched elastic strings in confining the motion of the conducting fluid to the neighbourhood of the sphere. Indeed, the situation would be similar to that of the ordinary fluid motion past a sphere fixed in a pipe, the magnetic lines of force playing the part of the wall of the pipe.

Now, it is well known in ordinary hydrodynamics that the two-dimensional flow of infinite extent cannot be treated by use of the Stokes approximation, but that it is applicable if the body is placed in a channel. Hence, it may be expected that the Stokes approximation would be useful for the two-dimensional flow if a uniform magnetic field is imposed on the flow, since the field will play the part of the bounding well.

In fact, Yosinobu and Kakutani[(2)] considered the two-dimensional Stokes flow of an electrically conducting fluid past a cylindrical body in a uniform magnetic field for two cases, one in which the undisturbed flow and magnetic field are parallel and the other in which they are perpendicular to each other. Their analysis is similar to Chester's. It was found that the cylinder is accompanied by two wake regions of roughly parabolic shape such that the vorticity and the induced electric current are confined there. The wakes extend in both directions parallel to the undisturbed magnetic field. The case of a circular cylinder was discussed in detail, and the formulae for the drag were obtained in the forms:

$$D_{\parallel} = \frac{8\pi p\nu U}{2\Omega+1}\left\{1-\frac{4\Omega+1}{2\Omega+1}\left(\frac{H}{4}\right)^2+0(H^4)\right\} \tag{2.9}$$

$$D_{\perp} = \frac{8\pi\varrho\nu U}{2\Omega+1}\left\{1-\frac{8\Omega^2+4\Omega+1-2(2\Omega-1)^2(\sigma_b/\sigma)}{2\Omega-1}\left(\frac{H}{4}\right)^2+0(H^4)\right\} \tag{2.10}$$

Here $D_{\parallel}$ and $D_{\perp}$ are the drags for the parallel and perpendicular fields respectively, and σ_b is the electrical conductivity of the cylinder. Also $H = (\sigma/\varrho\nu)^{1/2}\, B_\infty a$ is the Hartmann number, and

$$\Omega = -\{\gamma+\log(H/4)\} \tag{2.11}$$

$\gamma = 0{\cdot}57721\ldots$ being Euler's constant.

It is remarkable that the drag for the case of perpendicular field is dependent on the electrical conductivity σ_b of the cylinder and is always greater than that for the case of parallel field, which is in dependent of σ_b.

Several years ago, the author[(3)] proposed a new method of solving Oseen's equation for the two-dimensional flow, by use of complex variables. A similar method can be developed for treating the Stokes approximation for two-dimensional magnetohydrodynamic flow, on account of the similarity of the governing equations. By this method, Hasimoto[(4)] has obtained a very interesting result that, for the case of parallel fields at very small Hartmann number $H \ll 1$, the force acting on the cylinder is equal to that for the Oseen flow of ordinary fluid at the Reynolds number $R = H$. For example, the drag of a circular cylinder at Reynolds number R is given by[(3)]

$$D = \frac{8\pi\varrho\nu U}{2S+1}\left\{1-\frac{R^2}{8}\left(S+\frac{5}{8}\,\frac{1}{2S+1}\right)+0(R^4)\right\} \tag{2.12}$$

where

$$S = \log(4/R)-\gamma, \qquad R = aU/\nu \tag{2.13}$$

It will be seen that the first term of $D_{\parallel}$ can be obtained from D by replacing R by H. (It may be noted that the same relation holds for the case of a sphere. That is,

$$D = D_S\left\{1+\frac{3}{8}R+0(R^2)\right\}$$

which should be compared with Chester's result (2.1).)

The two-dimensional and axisymmetric flows are very simple in the sense that the electric field can be easily eliminated from the basic equations; indeed it is zero or constant. But in the general three-dimensional case, Chester's analysis cannot be followed in a straightforward manner. Gotoh[(5)] and the present author[(6)] developed independently methods for treating such cases. Gotoh's method may be regarded rather as a direct generalization of Chester's original method, In fact, it is based on the assumption that the imposed magnetic field is not disturbed by the motion of the body. But the electric field $\boldsymbol{E}$ (which was constant in Chester's method) is taken to be variable. Thus the basic equations are written in the non-dimensional form as

$$\left.\begin{aligned} \Delta v_x - H^2\left(\frac{\partial \Phi}{\partial y} + v_x\right) &= \frac{\partial p}{\partial x}, \\ \Delta v_y + H^2\left(\frac{\partial \Phi}{\partial x} - v_y\right) &= \frac{\partial p}{\partial y}, \\ \Delta v_z &= \frac{\partial p}{\partial z}, \\ \Delta \Phi - \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) &= 0, \end{aligned}\right\} \tag{2.14}$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, $\boldsymbol{v}(v_x, v_y, v_z)$ is the velocity, p the pressure, and Φ the electric potential:

$$\boldsymbol{E} = -\operatorname{grad} \Phi \tag{2.15}$$

Here the z-axis is chosen parallel to the undisturbed magnetic field.

Gotoh applied his method to the case of a sphere moving in an arbitrary direction with respect to the imposed uniform magnetic field. After elaborate manipulation of the modified Bessel functions and associated Legendre functions, he obtained the formulae for the force $\boldsymbol{F}$:

$$\left.\begin{aligned} F_x &= 6\pi a \varrho \nu U \sin\alpha \left\{1 + \frac{9}{16} H + 0(H^2)\right\}, \\ F_y &= 0, \\ F_z &= 6\pi a \varrho \nu U \cos\alpha \left\{1 + \frac{3}{8} H + 0(H^2)\right\}, \end{aligned}\right\} \tag{2.16}$$

where the undisturbed flow velocity is $U(\sin\alpha, 0, \cos\alpha)$. The conductivity of the sphere does not affect the force up to the order $0(H)$.

The present author's procedure is to start from the Oseen type approximation (to be discussed in the following section) and consider the limiting case in which the viscous magnetic Reynolds numbers R, R_m tend to zero. Thus putting

$$\boldsymbol{V} = U(\boldsymbol{e}'+\boldsymbol{v}), \qquad \boldsymbol{B} = B_\infty(\boldsymbol{e}+R_m\boldsymbol{b}) \tag{2.17}$$

where $\boldsymbol{e}$, $\boldsymbol{e}'$ are arbitrary unit vectors, and treating $\boldsymbol{v}$ and $\boldsymbol{b}$ as dependent variables, we have the Oseen approximation. Then, as its limiting case it was found that the Stokes approximation can be generally expressed in the form:

$$\left.\begin{aligned}
\boldsymbol{v} &= \frac{1}{2}(\boldsymbol{u}_1+\boldsymbol{u}_2), \\
\boldsymbol{b} &= \frac{\partial}{\partial x}\operatorname{grad}\phi + \frac{1}{2H}(\boldsymbol{u}_1-\boldsymbol{u}_2), \\
\frac{p}{\varrho\nu U} &= \bar{p} - H^2 b_x = \frac{H}{2}(\boldsymbol{u}_{2x}-\boldsymbol{u}_{1x}), \\
\bar{p} &= H^2\frac{\partial^2\phi}{\partial x^2},
\end{aligned}\right\} \tag{2.18}$$

where $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ and ϕ satisfy the equations:

$$\begin{aligned}
\left(\Delta \pm H\frac{\partial}{\partial x}\right)\boldsymbol{u}_{1,2} &= 0, \\
\Delta\phi &= 0
\end{aligned} \tag{2.19}$$

with $\boldsymbol{e} = (1, 0, 0)$. Here $\bar{p}$ is the (non-dimensional) total pressure, that is, the sum of the pressure and the magnetic pressure. Further, explicit expressions for $\boldsymbol{v}$, $\boldsymbol{b}$ and $\bar{p}$ have been obtained for an arbitrary three-dimensional body as

$$\left.\begin{aligned}
\boldsymbol{v} = \frac{1}{2}\sum\frac{\partial^{m+n}}{\partial y^m\partial z^n}\Bigg[&\left\{\left(\boldsymbol{C}_{mn}^{(0)}+\boldsymbol{C}_{mn}^{(1)}\frac{\partial}{\partial x}\right)\frac{e^{-k(r+x)}}{r}\right.\\
&\left.+\left(\boldsymbol{C}_{mn}^{\prime(0)}+\boldsymbol{C}_{mn}^{\prime(1)}\frac{\partial}{\partial x}\right)\frac{e^{-k(r-x)}}{r}\right\}\\
&-\frac{1}{k}D_{mn}^{(0)}\left(0,\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)g_e\Bigg],\\
b = \frac{1}{4k}\sum\frac{\partial^{m+n}}{\partial y^m\partial z^n}\Bigg[&\left\{\left(\boldsymbol{C}_{mn}^{(0)}+\boldsymbol{C}_{mn}^{(1)}\frac{\partial}{\partial x}\right)\frac{e^{-k(r-z)}}{r}\right.\\
&\left.-\left(\boldsymbol{C}_{mn}^{\prime(0)}+\boldsymbol{C}_{mn}^{\prime(1)}\frac{\partial}{\partial x}\right)\frac{e^{-k(r-x)}}{r}\right\}\\
+\frac{1}{k}\left(0,\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)&\left(D_{mn}^{(0)}g_0+D_{mn}^{(1)}\frac{1}{r}\right)+\frac{\partial^2\phi}{\partial x^2}\boldsymbol{e}\Bigg],\\
\bar{p} = 4k^2\frac{\partial^2\phi}{\partial x^2} = \sum&\frac{\partial^{m+n}}{\partial y^m\partial z^n}\left(D_{mn}^{(0)}+D_{mn}^{(1)}\frac{\partial}{\partial x}\right)\frac{1}{r},
\end{aligned}\right\} \tag{2.20}$$

where $k = H/2$, $\boldsymbol{e} = (1, 0, 0)$, and

$$\left.\begin{aligned} g_e &= \tfrac{1}{2}\{E(k(r+x))+E(k(r-x))-\log(y^2+z^2)\}, \\ g_0 &= \tfrac{1}{2}\left\{E(k(r-x))-E(k(r+x))-\log\frac{r-x}{r+x}\right\}, \end{aligned}\right\} \tag{2.21}$$

and

$$E(\xi) = \int_{\infty}^{\xi} \frac{e^{-\xi}}{\xi}\,\mathrm{d}\xi = \gamma+\log\xi+\sum_{n=1}^{\infty}\frac{(-1)^n\xi^n}{n\cdot n!} \tag{2.22}$$

γ being Euler's constant. The arbitrary constants $\boldsymbol{C}_{mn}^{(0)}$ $(A_{mn}^{(0)}, B_{mn}^{(0)}, C_{mn}^{(0)})$, $D_{mn}^{(0)}$ etc. are not independent because of the conditions div $\boldsymbol{V} = 0$, div $\boldsymbol{b} = 0$, but are subject to the relations such as

$$\left.\begin{aligned} &D_{00}^{(0)} = 0, \\ &A_{00}^{(0)} = 2kA_{00}^{(1)}, \quad A_{00}^{\prime(0)} = -2kA_{00}^{\prime(1)}, \\ &B_{00}^{(0)} = B_{00}^{\prime(0)} = -D_{10}^{(0)}, \quad C_{00}^{(0)} = C_{00}^{\prime(0)} = -D_{01}^{(0)} \end{aligned}\right\} \tag{2.23}$$

From (2.20) it is obvious that the flow field contains two wake regions. Namely, the terms containing $\exp(-k(r+x))$ or $E(k(r+x))$ represent a paraboloidal wake extending in the direction of the negative x-axis, while those containing $\exp(-k(r-x))$ or $E(k(r-x))$ a wake around the positive x-axis. The outer region is only influenced by the $D_{mn}^{(0)}$ terms, and has a character of two-dimensional irrotational motion, since the flow there is given by

$$\left(0, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\frac{1}{4k}\sum D_{mn}^{(0)}\frac{\partial^{m+n}}{\partial y^m \partial z^n}\log(y^2+z^2)$$

which represents a uniform distribution of various kinds of multiplets on the x-axis.

Further, the formulae for the force $\boldsymbol{F}$ and moment $\boldsymbol{M}$ acting on the body have been obtained:

$$\left.\begin{aligned} F_x &= -4\pi\varrho\nu U(A_{00}^{(0)}+A_{00}^{\prime(0)}+D_{00}^{(1)}), \\ F_y &= 4\pi\varrho\nu U D_{10}^{(0)}, \\ F_z &= 4\pi\varrho\nu U D_{01}^{(0)}; \end{aligned}\right\} \tag{2.24}$$

$$\left.\begin{aligned} M_x &= 2\pi\varrho\nu(UC_{10}^{(0)}+C_{10}^{\prime(0)}-B_{10}^{(0)}-B_{01}^{\prime(0)}), \\ M_y &= 2\pi\varrho\nu U\{D_{01}^{(1)}+2(A_{01}'+A_{01}^{\prime(0)})\}, \\ M_z &= -2\pi\varrho\nu U\{D_{10}^{(1)}+2(A_{10}^{(0)}+A_{10}^{\prime(0)})\}. \end{aligned}\right\} \tag{2.25}$$

As an example of application, the author has treated an arbitrary slow motion of a sphere. On account of the linearity of the Stokes approximation and the symmetry of the geometry, it is sufficient to consider the four special cases:

(i) translation parallel to the x-axis,

(ii) translation parallel to the y-axis,

(iii) rotation about the y-axis

(iv) rotation about the y-axis.

It has been found that the force and moment acting on the sphere are given, in each of the above-mentioned cases, by

$$\left.\begin{aligned} &\text{(i)}\ F_z = 6\pi\left(1+\frac{3}{4}k\right)\varrho\nu Ua, \\ &\text{(ii)}\ F_y = 6\pi\left(1+\frac{9}{8}k\right)\varrho\nu Ua, \\ &\text{(iii)}\ M_x = -8\pi\left(1+\frac{4}{15}k^2\right)\varrho\nu\Omega a^3, \\ &\text{(iv)}\ M_0 = -8\pi\left(1+\frac{4}{45}k^2\right)\varrho\nu\Omega a^3, \end{aligned}\right\} \tag{2.26}$$

all the other components of force and moment vanishing. Here U and Ω are the magnitude of linear and angular velocities, respectively. The results (i) and (ii) are in agreement with those of Chester (2.1) and of Gotoh (2.16), respectively.

It may be mentioned that quite recently Takaisi[(7)] has considered the steady slow motion of a circular cylinder in a semi-infinite mass of a viscous, electrically conducting fluid parallel to its bounding plane wall in the presence of a uniform magnetic field. Using Stokes approximation, he has carried out detailed calculations for two cases: parallel and perpendicular magnetic fields.

3. THE OSEEN TYPE APPROXIMATION

The above-mentioned Stokes approximation corresponds to the assumption of vanishingly small Reynolds number R and magnetic Reynolds number R_m with finite Hartmann number H. Accordingly, if the pressure number S and the Alfvén number A are defined by

$$S = 1/A^2 = H^2/RR_m \tag{3.1}$$

we have $S \to \infty$ and $A \to 0$ for Stokes approximation.

Now, the Oseen type approximation can be developed similarly to the ordinary fluid dynamics, on the basis of the method of small disturbances. Thus, assuming that

$$\boldsymbol{V} = U(\boldsymbol{e}'+\boldsymbol{v}), \qquad \boldsymbol{B} = B_\infty(\boldsymbol{e}+R_m\boldsymbol{b}) \tag{3.2}$$

where $\boldsymbol{e}'$, $\boldsymbol{e}$ are unit vectors, and $\boldsymbol{v}$, $\boldsymbol{b}$ are small perturbations, the basic equations for magnetohydrodynamics can be linearized with respect to $\boldsymbol{v}$ and $\boldsymbol{b}$.

Yosinobu[8] and Gotoh[9] considered the case of parallel fields: $\boldsymbol{e} = \boldsymbol{e}'$. Yosinobu made a detailed investigation for the case in which the electric field vanishes everywhere. This is true for two-dimensional and axisymmetric flows. As a concrete example, he treated the two-dimensional flow past a circular cylinder. His result for the drag is expressed as

$$D = \frac{16\pi\varrho\nu U(k_1-k_2)}{(R_m-2k_2)\{1+2\Omega(|k_2|)\}-R-2k_1)\{1+2\Omega(k_1)\}} \tag{3.3}$$

where

$$k_{1,2} = \tfrac{1}{4}\{(R+R_m)\pm\sqrt{(R-R_m)^2+4H^2}\} \tag{3.4}$$

$$\Omega(k) = -\{\gamma+\log(k/2)\}, \quad \gamma = 0{\cdot}57721... \tag{3.5}$$

$$R = \frac{aU}{\nu}, \quad R_m = \sigma\mu aU \tag{3.6}$$

$$H = (\sigma/\varrho\nu)^{1/2}B_\infty a \tag{3.7}$$

(i) The case: $R_m = 0$ or $S = 0$.

In this case (3.3) becomes

$$D = \frac{8\pi\varrho\nu U}{1+2\Omega(R/2)} \tag{3.8}$$

This is Lamb's well-known result for ordinary viscous flow.

(ii) The case: $R = R_m = 0$, $H =$ finite.

Here, (3.3) becomes

$$D = \frac{8\pi\varrho\nu U}{1+2\Omega(H/2)} \tag{3.9}$$

This agrees, to the first approximation, with the result of Yosinobu and Kakutani (2.9) based on the Stokes approximation.

It is interesting to note that near $S = 1(H_2 \fallingdotseq RR_m)$, we have

$$D \sim \frac{\varrho\nu U}{\log|S-1|} \tag{3.10}$$

so that the drag has a sharp minimum zero at $S = 1$. This seems to imply that the Oseen approximation fails near $S = 1$ or $A = 1$.

Similar investigations have been made for the case of a sphere by Gotoh[9]. In this case also the drag behaves differently according as S is smaller or larger than 1. Thus

$$D = 6\pi\varrho\nu Ua\left[1+\frac{3}{8}R-\left\{\frac{19}{320}R^2+\frac{2H^2}{5(2+\varkappa)}\right\}+0(\delta^3)\right], \quad S<1 \tag{3.11}$$

$$D=6\pi\varrho\nu Ua\left[1+\frac{3}{8}\frac{R^2-RR_m+2H^2}{8\{(R-R_m)^2+4H^2\}^{1/2}}\right.$$
$$-\frac{19R^2(R+R_m)^2+(H^2RR_m)(76R^2-180H^2)}{320\{(R-R_m)^2+4H^2\}}$$
$$\left.-\frac{2H^2}{5(2+\varkappa)}+0(\delta^3)\right],\qquad S>1 \tag{3.12}$$

where $\varkappa=\mu_b/\mu$ is the ratio of the permeabilities of the sphere and the fluid, and k_1 and k_2 have been assumed to be small of $0(\delta)$. (Gotoh has obtained the formulae (3.11) and (3.12) correct up to $0(\delta^3)$). The above formulae give interesting limiting cases.

(i) The case: $R_m=0$ or $S=0$, so that $H=0$ and $R\neq 0$.
Here (3.11) becomes

$$\frac{D}{6\pi\varrho\nu Ua}=1+\frac{3}{8}R-\frac{19}{320}R^2+\frac{71}{2560}R^3+0(R^4) \tag{3.13}$$

This is Goldstein's result.

(ii) The case: $R=R_m=0,\ H\neq 0(S\to\infty),\ \varkappa=1$
In this case (3.12) gives Chester's result. (2.1).

(iii) The case: $S=1$.
Both (3.11) and (3.12) give

$$\frac{D}{6\pi\varrho\nu Ua}=1+\frac{3}{8}R-\left\{\frac{19}{320}R^2+\frac{2RR_m}{5(2+\varkappa)}\right\}+0(\delta^3) \tag{3.14}$$

Thus, contrary to the case of two-dimensional flow, the drag has a finite value (which is minimum) at $S=1$.

It is interesting to note that if $0(\delta^2)$ is neglected, the drag is independent of H and hence of the magnetic field as long as $S<1$, while for $S>1$ it increases with the magnetic field.

The present author[6] has considered the linearization of the unsteady three-dimensional motion of an incompressible, viscous and electrically conducting fluid. The basic equations can be expressed as

$$\varrho\frac{\partial \boldsymbol{V}}{\partial t}=-\varrho(\boldsymbol{V}\cdot\text{grad})\,\boldsymbol{V}+\frac{1}{\mu}(\boldsymbol{B}\cdot\text{grad})\boldsymbol{B}-\text{grad}\left(\varrho+\frac{B^2}{2\mu}\right)+\varrho\nu\Delta\boldsymbol{V} \tag{3.15}$$

$$\frac{\partial \boldsymbol{B}}{\partial t}=\text{curl}(\boldsymbol{V}\times\boldsymbol{B})-\frac{1}{\sigma\mu}\text{curl}\,\text{curl}\boldsymbol{B}, \tag{3.16}$$

$$\text{div}\,\boldsymbol{V}=0 \tag{3.17}$$

$$\text{div}\,\boldsymbol{B}=0 \tag{3.18}$$

Here ϱ, μ, σ, ν are assumed to be constant.

Substituting (3.2) and neglecting second order terms with respect to $\boldsymbol{v}$ and $\boldsymbol{b}$, we have, after some calculations,

$$\left.\begin{aligned} \boldsymbol{v} &= \boldsymbol{v}_0+\boldsymbol{v}_1\,, \qquad \boldsymbol{b} = \boldsymbol{b}_0+\boldsymbol{b}_1\,, \\ p &= \frac{\varrho\nu U}{l}(\bar{p}-H^2 b_x)\,, \end{aligned}\right\} \tag{3.19}$$

where

$$\left.\begin{aligned} \boldsymbol{v}_0 &= R_m\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial s}\right)\operatorname{grad}\phi\,, \\ \boldsymbol{b}_0 &= \frac{\partial}{\partial x}\operatorname{grad}\phi\,, \\ \bar{p} &= \left\{H^2\frac{\partial^2}{\partial x^2}-RR_m\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial s}\right)^2\right\}\phi\,, \\ \Delta\phi &= 0\,; \end{aligned}\right\} \tag{3.20}$$

$$\left.\begin{aligned} &\left\{\Delta-R\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial s}\right)\right\}\boldsymbol{v}_1 = -H^2\frac{\partial \boldsymbol{b}_1}{\partial x}\,, \\ &\left\{\Delta-R_m\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial s}\right)\right\}\boldsymbol{b}_1 = -\frac{\partial \boldsymbol{v}_1}{\partial x}\,, \\ &\operatorname{div}\boldsymbol{v}_1 = 0\,, \qquad \operatorname{div}\boldsymbol{b}_1 = 0\,; \end{aligned}\right\} \tag{3.21}$$

$$\boldsymbol{e}\cdot\operatorname{grad} \equiv \frac{\partial}{\partial x}, \qquad \boldsymbol{e}'\cdot\operatorname{grad} \equiv \frac{\partial}{\partial s} \tag{3.22}$$

Here (3.20) and (3.21) are expressed in non-dimensional forms such that $\boldsymbol{r}(x, y, z)$ and t are used in place of $\boldsymbol{r}/l$ and $(U/l)t$. From the above equations, we can derive various special cases For steady flow, we have simply to put $\partial/\partial t \equiv 0$.

Elimination of $\boldsymbol{b}_1$(or $\boldsymbol{v}_1$) from (3.21) gives a fourth order differential equation for $\boldsymbol{v}_1$(or $\boldsymbol{b}_1$). But it can easily be seen that for two special cases (i) $R = R_m$ and (ii) $\boldsymbol{e} = \boldsymbol{e}'$, the equation can be reduced to a pair of second order equations of the Oseen type.

(i) The case: $R = R_m$

$$\boldsymbol{v}_1 = \frac{1}{2}(\boldsymbol{u}_1+\boldsymbol{u}_2), \qquad \boldsymbol{b}_1 = \frac{1}{2H}(\boldsymbol{u}_1-\boldsymbol{u}_2) \tag{3.23}$$

$$\left(\Delta-H_i\frac{\partial}{\partial x^{(i)}}\right)\boldsymbol{u}_i = 0, \qquad \mathrm{i} = 1,2 \tag{3.24}$$

where $x^{(i)}$ is the coordinate axis whose direction is given by the unit vector $\boldsymbol{e}_i$ such that

$$H_i\boldsymbol{e}_i = R\boldsymbol{e}'\mp H\boldsymbol{e} \tag{3.25}$$

(Here and below, the upper and lower signs correspond to $i = 1$ and 2, respectively). Hence

$$H_i = H^2\sqrt{1+A^2 \mp 2A\cos\alpha} \tag{3.26}$$

$$\tan\Theta_i = \frac{A\sin\alpha}{A\cos\alpha \mp 1} \tag{3.27}$$

where

$$\boldsymbol{e}\cdot\boldsymbol{e}' = \cos\alpha, \quad \boldsymbol{e}\cdot\boldsymbol{e}_i = \cos\Theta_i$$

From (3.24) it can be seen that two wake regions of paraboloidal shap W_1, W_2 appear around the $x^{(1)}$ and $x^{(2)}$ axes with the widths proportional to $H_1^{-1/2}$ and $H_2^{-1/2}$, respectively.

(ii) The case of parallel fields at infinity: $\boldsymbol{e} = \boldsymbol{e}'$

In this case we have

$$\boldsymbol{u}_1 = \boldsymbol{v}_1 + \lambda_1 \boldsymbol{b}_1, \quad i = 1,2 \tag{3.28}$$

with

$$\left(\Delta - R_i\frac{\partial}{\partial x}\right)\boldsymbol{u}_i = 0 \tag{3.29}$$

$$R_i = \tfrac{1}{2}(R+R_m) \mp \sqrt{\tfrac{1}{4}(R-R_m)^2 + H^2} \tag{3.30}$$

$$\lambda_i = \tfrac{1}{2}(R-R_m) \pm \sqrt{\tfrac{1}{4}(R-R_m)^2 + H^2} \tag{3.31}$$

Eq. (3.29) shows that here also two paraboloidal wakes W_1, W_2 appear along the x-axis. It is readily seen that $R_1 \gtreqless 0$ according as $A \gtreqless 0$, whereas always $R_2 > 0$. Hence the wake W_2 is always situated along the positive x-axis, while the wake W_1 appears around the positive or negative x-axis according as the Alfvén number A is larger or smaller than 1. There does not seem to exist such a simple asymptotic behaviour of fluid flow except for the above-mentioned special cases.

4. STEADY MOTION OF A COMPRESSIBLE, INVISCID, PERFECTLY CONDUCTING FLUID WITH PARALLEL VELOCITY AND MAGNETIC FIELDS

The small-perturbation theory of magnetoaerodynamics was first considered by Sears and his group[10], [11]. In particular, Resler[12] established the basic equation for the two-dimensional motion of a perfectly conducting, inviscid, compressible fluid with parallel velocity and magnetic fields in a very elegant form analogous to Prandtl–Glauert's equation. The same equation had also been found by Taniuti[13] without the asumption of small perturbation. The present author[14] first obtained Resler's equation for the three-dimensional case by using the small-perturbation theory, and then without making such an approximation[6].

The steady flow of an inviscid, perfectly conducting, compressible fluid is governed by the equations:

$$\operatorname{div} \varrho \boldsymbol{V} = 0 \tag{4.1}$$

$$\boldsymbol{V} \times \boldsymbol{w} - \frac{1}{\varrho} \boldsymbol{B} \times \boldsymbol{j} = \operatorname{grad}\left(\frac{q^2}{2} + P + \Omega\right) \tag{4.2}$$

$$\boldsymbol{V} \times \boldsymbol{B} = -\boldsymbol{E} = \operatorname{grad} \Phi \tag{4.3}$$

$$\operatorname{div} \boldsymbol{B} = 0 \tag{4.4}$$

where

$$\boldsymbol{w} = \operatorname{curl} \boldsymbol{V} \tag{4.5}$$

$$\boldsymbol{j} = \frac{1}{\mu} \operatorname{curl} \mathrm{B} \tag{4.6}$$

$$P = \int \frac{\mathrm{d}p}{\varrho} \tag{4.7}$$

Ω is the potential of the extential force (such as gravity), Φ is the electric potential, and the density p is assumed to be a definite function of the pressure p only.

It can readily be shown that Φ is constant along each streamline and each magnetic line of force. Therefore, if the flow velocity and magnetic field are parallel to each other at infinity upstream: $\boldsymbol{V}_\infty // \boldsymbol{B}_\infty$, then $\boldsymbol{E}_\infty = -\boldsymbol{V}_\infty \times \boldsymbol{B}_\infty = 0$ and hence Φ are constant (so that $\boldsymbol{V}//\boldsymbol{B}$) everywhere in the field of flow. Then analogously to the Bernoulli theorem in the conventional fluid dynamics, $q^2/_2 + P + \Omega$ is constant along each Streamline. After some calculation, it can be shown that $\boldsymbol{V}$ and $\boldsymbol{B}$ are expressed in terms of a certain vector $\boldsymbol{b}$:

$$\frac{\boldsymbol{V}}{U} = \frac{1-A_\infty^{-2}}{1-A^{-2}} \boldsymbol{b}, \qquad \frac{\boldsymbol{B}}{B_\infty} = \frac{1-A_\infty^2}{1-A^2} \boldsymbol{b} \tag{4.8}$$

where

$$\operatorname{div} \tau \boldsymbol{b} = 0, \qquad \operatorname{curl} \boldsymbol{b} = 0 \tag{4.9}$$

with

$$\tau = \frac{1-A_\infty^2}{1-A^2} \tag{4.10}$$

$$A^2 = q^2/V_a^2 = A_\infty^2\, \varrho_\infty/\varrho, \qquad V_a = B/\sqrt{\mu\varrho} \tag{4.11}$$

A, V_a being the local Alfvén number and Alfvén velocity, respectively. If there is no external force, $\Omega = 0$. Then, in view of the generalized Bernoulli theorem, all the physical quantities q, B etc. are functions of a single physical quantity. Therefore τ is a definite function of $b = |\boldsymbol{b}|$. Hence (4.9) can be regarded as a pair of equations governing the irrotational flow of a hypothetical compressible fluid, $\boldsymbol{b}$ and τ playing the parts of velocity and density. Thus, the steady magnetohydrodynamic flow of an inviscid, perfectly conducting, compressible fluid starting from a uniform state with parallel velocity and magnetic fields at infinity can be reduced to the conventional gas dynamics of a hypothetical gas. For example, the equations of motion can be obtained simply by using the method of conventional gas dynamics:

$$(a^2-b_x^2)\frac{\partial^2\phi}{\partial x^2}+(a^2-by^2)\frac{\partial^2\phi}{\partial y_2}+(a^2-bz^2)\frac{\partial^2\phi}{\partial z^2}$$

$$-2b_yb_z\frac{\partial^2\phi}{\partial y\partial z}-2bzbx\frac{\partial^2\phi}{\partial z\partial x}-2bxby\frac{\partial^2\phi}{\partial x\partial y}=0 \tag{4.12}$$

where

$$\boldsymbol{b}=\operatorname{grad}\phi, \tag{4.13}$$

$$a^2=\frac{A^2+M^2-1}{A^2M^2}b^2 \tag{4.14}$$

Here M is the local Mach number defined by

$$M=\frac{q}{c},\qquad c^2=\frac{\mathrm{d}p}{\mathrm{d}\varrho} \tag{4.15}$$

a plays the part of local sound speed for the hypothetical gas and so may be called the pseudo-sound velocity. Thus, the pseudo-Mach number may be defined by

$$m=b/a \tag{4.16}$$

For small perturbation, we may put $\boldsymbol{b}\fallingdotseq(1,0,0)$. Then

$$a^2\fallingdotseq\frac{A_\infty^2+M_\infty^2-1}{A_\infty^2M_\infty^2}=\frac{1}{m_\infty^2}$$

so that (4.12) is reduced to

$$(1-m_\infty^2)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0 \tag{4.17}$$

which is Resler's equation, originally obtained for the two-dimensional flow[12].

It should be remarked that analogy between the magnetohydrodynamics and the ordinary gas dynamics of a hypothetical gas can be used to develop a method of calculation similar to von Kármán–Tsien's hodograph method. Moreover, it should be emphasized that the method is applicable to the general three-dimensional problems, in contrast to the fact that von Kármán–Tsien's method is applicable to the two-dimensional problem only.

5. SMALL-PERTURBATION THEORY OF AN INVISCID COMPRESSIBLE FLUID OF FINITE ELECTRICAL CONDUCTIVITY

Since the Reynolds number of the flow is usually very large, the fluid can be regarded as inviscid, while compressibility may be important for flow at high speeds. Small-perturbation theory of such flows has also been considered in Japan. Ando[15] considered the general three-dimensional flow of an ideal gas in the presence of a uniform magnetic field at an arbitrary orientation with respect to the undisturbed uniform flow. He obtained the basic equations in the forms:

$$\boldsymbol{V} = U(\boldsymbol{e}+\boldsymbol{v}), \qquad \boldsymbol{B} = B_\infty(\boldsymbol{e}'+\boldsymbol{b}) \tag{5.1}$$

with

$$\left.\begin{aligned} \boldsymbol{v} &= \left[\left(\Delta - R_m\frac{\partial}{\partial x}\right)\frac{\partial}{\partial x}\,\mathrm{grad} + \frac{R_m}{A^2}\,\boldsymbol{e}'\frac{\partial}{\partial \eta}\Delta\right)\Phi, \\ \boldsymbol{b} &= R_m\left(\boldsymbol{e}'\Delta - \frac{\partial}{\partial\eta}\,\mathrm{grad}\right)\frac{\partial\Phi}{\partial x}, \\ \frac{p-p_\infty}{\varrho_\infty U^2} &= -\left\{\left(\Delta - R_m\frac{\partial}{\partial x}\right)\frac{\partial}{\partial x} + \frac{R_m}{A^2}\Delta\right\}\frac{\partial\Phi}{\partial x}, \end{aligned}\right\} \tag{5.2}$$

where Φ satisfies the equation:

$$\left\{\left(\Delta - R_m\frac{\partial}{\partial x}\right)\left(\Delta - M_\infty^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial}{\partial x} - \frac{R_m}{A_\infty^2}\left(M_\infty^2\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial\eta^2}\right)\Delta\right\}\Phi = 0 \tag{5.3}$$

Here $\boldsymbol{e} = (1, 0, 0)$, $\boldsymbol{e}$ are unit vector and $\partial/\partial\eta \equiv \boldsymbol{e}'\cdot\mathrm{grad}$. Also, M_∞ and A_∞ are the Mach number and Alfvén number of the undisturbed flow:

$$M_\infty = U/C_\infty, \qquad A_\infty = U/(B_\infty/\sqrt{\mu\varrho_\infty}) \tag{5.4}$$

It is easy to deduce Resler's equation (4.17) from (5.3), by putting $\boldsymbol{e}' = \boldsymbol{e}$ and $R_m \to \infty$. As an example, Ando treated the flow along a sinusoidal wall of non-magnetic and non-conducting material, for the case in which the magnetic field ($\boldsymbol{e}'$) makes an arbitrary angle α with the wall. The results are of course in agreement with those by Sears and Resler[11] for the special cases of $\alpha = 0$ and $\pi/2$.

Sakurai[16], [17] independently considered the two-dimensional flow of an ideal gas at small magnetic Reynolds number. In particular, he treated the flow past a thin symmetric aerofoil at zero angle of attack, in the presence of a uniform magnetic field making a small angle δ with the undisturbed flow. Thus he found the formulae for the lift and drag:

$$C_L = 0 \tag{5.5}$$

$$C_D = -\frac{R_m t^2}{\pi A_\infty^2\sqrt{1-M_\infty^2}}\int_{-1}^{1}\int_{-1}^{1} g'(\xi)g'(\xi_1)\log|\xi-\xi_1|\,\mathrm{d}\xi\,\mathrm{d}\xi_1 \tag{5.6}$$

for $M_\infty < 1$, and

$$C_L = -2t\frac{R_m}{A_\infty^2}\delta\int_{-1}^{1} g(\xi)\,\mathrm{d}\xi \tag{5.7}$$

$$C_D = \frac{2t^2}{\sqrt{M_\infty^2-1}}\int_{-1}^{1}[g'(\xi)]^2\,\mathrm{d}\xi \tag{5.8}$$

for $M_\infty > 1$. Here the aerofoil is represented by

$$y = \pm\tan(x) \tag{5.9}$$

Quite recently, Ando[18] has extended his theory to cover the case of unsteady flow and also gave special considerations to the case of "weak interaction" ($R_m \to 0$ and $R_m/A_\infty^2 \ll 1$) for steady flows past two-dimensional symetric aerofoils and axisymmetric slender bodies.

6. RAYLEIGH'S PROBLEM

Consider a semi-infinite fluid bounded by an infinite flat plate. What will be the motion of the fluid if the plate is suddenly set in motion parallel to itself at a uniform speed? This is known as Rayleigh's problem. The magnetohydrodynamical version of this problem was first considered by Rossow[19]. The problem is important and interesting because it furnishes the exact solution of magnetohydrodynamics. As a matter of fact, if the x-axis is taken in the plane of the plate and parallel to its motion, the velocity, magnetic induction, electric field and electric current density can be expressed, respectively, as

$$\boldsymbol{V} = (u,\ 0,\ 0), \quad \boldsymbol{B} = (B_0,\ B_0,\ 0)$$
$$\boldsymbol{E} = (0,\ 0,\ E), \quad \boldsymbol{j} = (p,\ 0,\ j)$$

where B_0 is the undisturbed uniform magnetic field perpendicular to the plate.

Rossow's analysis is approximate in the sense that he assumed (essentially) $E =$ constant at any instant[14], p. 37. Hasimoto[20] made rigorous analysis of the problem for the two cases: (i) non-conducting plate and (ii) perfectly conducting plate. The two cases differ in the boundary condition at the wall $y = 0$. Thus, $B_x = 0$ for the former and $\partial B_x/\partial y = 0$ for the latter. It was found that the frictional drag per unit area is given by

$$-\varrho\nu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\varrho\nu U}{\sqrt{\pi\nu t}}\begin{cases} e^{-mt} + \sqrt{\pi m t}\,\mathrm{erf}\,\sqrt{mt}, & \text{(i)} \\ e^{-mt} & \text{(ii)} \end{cases} \tag{6.1}$$

and the magnetic drag (due to Maxwell's stress) is

$$-\frac{B_0}{\mu}(Bx)_{y=0} = \begin{cases} 0 & \text{(i)} \\ \varrho V_a U \operatorname{erf} \sqrt{mt} & \text{(ii)} \end{cases} \tag{6.2}$$

where

$$\operatorname{erf} \eta = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-t^2}\, dt \tag{6.3}$$

$$m = \left(\frac{V_a}{\sqrt{\nu}+\sqrt{\nu_m}}\right)^2, \qquad V_a = \frac{B_0}{\sqrt{\varrho\mu}} \tag{6.4}$$

V_a, ν, $\nu_m = 1/\sigma\mu$ being the Alfvén velocity, the kinematic viscosity, and the magnetic viscosity, respectively.

Also, as $t \to \infty$, the flow tends to a steady state represented by

$$\left.\begin{aligned} \frac{u}{U} &= \frac{\sqrt{\nu}}{\sqrt{\nu}+\sqrt{\nu_m}}\left(1+\sqrt{\frac{\nu_m}{\nu}}\, e^{-Hy}\right), \\ \frac{B_x}{B_0} &= -\frac{A\sqrt{\nu}}{\sqrt{\nu}+\sqrt{\nu_m}}(1-e^{-Hy}), \end{aligned}\right\} \quad \text{(i)} \tag{6.5}$$

$$u = U, \qquad B_x = -AB_0 \qquad \text{(ii)} \tag{6.6}$$

where $H = V_a(\nu\,\nu_m)^{-1/2}$. ($Hy$ is the Hartmann number.)

i Further, simple expressions have been obtained for u and B at any instant for the special case $\nu = \nu_m$.

The flow of semi-infinite mass of fluid due to a harmonically oscillating flat plate can be dealt with in a similar manner to the Rayleigh problem. This problem was investigated by Kakutani both for the case of a non-conducting[21] and for a perfectly conducting plate[22]. He obtained exact solutions for arbitrary values of the Reynolds number R, the magnetic Reynolds number R_m and the magnetic pressure number S Here R, R_m and S are defined as

$$R = U^2/\nu\omega, \qquad R_m = U^2/\nu_m\omega, \qquad S = B_0^2/\varrho\mu U^2 = 1/A^2 \tag{6.7}$$

where U and ω are the amplitude and angular frequency of the velocity of the oscillating plate. It was found that the amplitude and the phase lag of the total drag (viscous stress+Maxwell's stress) are always increased by the effect of magnetic field. Further, it was shown that Rossow's assumption of $\boldsymbol{E} = 0$ corresponds to the "magnetic Stokes approximation", which consists essentially in assuming $R_m \ll 1$ and $SR_m = 0(1)$, and that the cases of magnetic field fixed to the plate and to the fluid correspond to the cases of a perfectly conducting and a non-conducting plate, respectively.

7. MOTION OF A MAGNETIZED BODY THROUGH VISCOUS CONDUCTING FLUID

Even in the absence of external magnetic field, some magnetohydrodynamical effect will appear when a magnetized body moves through an electrically conducting fluid. Tamada[23] studied such a problem for the case of a sphere and a circular cylinder carrying a magnetic dipole at their centres. The fluid was assumed to be incompressible, and both the viscous and magnetic Reynolds numbers R, R_m to be small.

In non-dimensional form, the basic equations can be expressed, for two-dimensional and axisymmetric flows, as

$$R(\boldsymbol{V}\cdot\text{grad})\,\boldsymbol{V} = -\text{grad}\,p + \Delta\boldsymbol{V} + H^2(\boldsymbol{V}\times\boldsymbol{B})\times\boldsymbol{B} \tag{7.1}$$

$$\text{curl}\,\boldsymbol{B} = R_m(\boldsymbol{V}\times\boldsymbol{B}) \tag{7.2}$$

$$\text{div}\,\boldsymbol{V} = 0 \tag{7.3}$$

$$\text{div}\,\boldsymbol{B} = 0 \tag{7.4}$$

Assuming R_m to be small, $\boldsymbol{B}$ can be found from (7.2) and (7.4) as

$$\boldsymbol{B} = \boldsymbol{B}_0 = \text{grad}\,\chi \tag{7.5}$$

χ being the magnetic potential. Introducing (7.5) into (7.1) and neglecting the left-hand side, we have Stokes approximation. For small values of the Hartmann number H the equation could be integrated by the method of perturbation. In particular, the drag of the sphere carrying a magnetic dipole (represented by $\chi = -r^{-2}\cos\theta$) was obtained in the form:

$$D = D_d + D_m = 6\pi\varrho\nu aU\left(1+\frac{2}{75}H^2\right) \tag{7.6}$$

with

$$D_d = 6\pi\varrho\nu aU\left(1-\frac{1}{25}H^2\right) \tag{7.7}$$

$$D_m = \frac{2}{5}\pi H^2\varrho\nu aU \tag{7.8}$$

where D_d and D_m are respectively the dynamic and magnetic drags.

Similar calculations were carried out for the case of a circular cylinder carrying a magnetic dipole with axis making an angle α with the direction of the motion. In this case the lift L and the drag D are given by

$$D = \frac{8\pi\varrho\nu U}{2\Omega+1-\dfrac{H^2}{48}(7-\cos 2\alpha)} \tag{7.9}$$

$$L = \frac{\pi H^2\varrho\nu U\sin 2\alpha}{6(4\Omega^2-1)} \tag{7.10}$$

where

$$\Omega = \log(4/R) - \gamma, \qquad R = a\,U/\nu$$

Further, the case of a circular cylinder carrying a concentric-circular magnetic field: $\chi = -\log r$ (produced by an electric current along its axis) was considered. The drag was calculated to be

$$D = \frac{8\pi\varrho\nu U}{2\Omega+1}\left\{1+\frac{1}{8}H^2(\log R_m)^2\right\} \tag{7.11}$$

All the above examples are common in showing that the loaded magnetic field retards the flow and reduces the dynamic stress on the body, but the magnetic drag always counteracts the reduction so that the total drag is always increased by the magnetohydrodynamical effect.

8. OTHER WORKS

In this paper we have surveyed the researches carried out in Japan concerning the magnetoaerodynamic flow past a body. There have also been researches dealing with boundary layers or jets. These will be only briefly mentioned in the following lines.

Sakurai(24) studied the two-dimensional and axisymmetric jets in the presence of a longitudinal magnetic field. Kakutani(25) studied the axisymmetric stagnation-point flow under transverse magnetic field. Tamada and Sone(26) dealt with the Blasius flow along a semi-infinite flat plate under parallel magnetic field. Yasuhara(27) obtained exact solutions of flows along a circular cylinder and a flat plate with uniform suction over the surface in the presence of transverse magnetic field. Mori(28) treated the laminar free-convention flow and heat transfer of electrically conducting fluid on a vertical flat plate in the presence of a transverse magnetic field, using boundary layer approximation. Quite recently, Morioka(29) has investigated the structure of the two-dimensional and the axisymmetric jets of a perfectly conducting inviscid gas with parallel magnetic field on the basic of small-perturbation theory.

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